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The Effects of Crowding and Toxicant on Biological Food - Chain System: A Mathematical Approach

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Abstract This work considers and studies a prey-predator mathematical system to explain the behavior and impacts of a food-chain system in the presence of toxicants and crowding. According to the system, only prey species defend themselves by releasing toxins. Using stability requirements, all possible equilibrium points of the system are discussed for local stability. It has been noted that when the toxicant or crowding effect is present, the system under consideration will survive. Lastly, numerical simulation is done to validate the analytical findings.

MSC: 93A30; 92D40; 34D20

Keywords: Crowding; Toxicant; Food-chain; Variational Matrix; Stability

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1. Introduction

Within an ecosystem, interactions occur between every living creature and its surroundings. An ecosystem's productivity, stability, and balance depend on continuing interactions. Predation, or the interaction between a predator and prey, is one type of interaction in an ecosystem, Molla et al. (2022) [8] and Manaqib et al. (2022) [4]. Restoring balance in the populations of predators and prey within an ecosystem is one of the objectives of this interaction. Predators keep prey populations in check, which harms the lower levels of the food chain. A predator, on the other hand, will run out of food to survive without prey.

Many species in the ecosystem have been driven to extinction by unchecked and unethical emissions of toxins into the environment. Prey predator species also face significant threats to their existence and may go extinct entirely if the situation is not controlled. Marine populations accumulate the majority of toxicants or pollutants and are then transferred to other populations of prey predators throughout the food chain, potentially affecting higher trophic levels. Fascinatingly, in the habitat of prey and predator, animals employ various tactical methods or strategies to defend themselves, employing toxins as their weapons. These species discharge toxicants or pollutant compounds to preserve

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their habitat or populations. Thus, the impact of harmful substances on prey-predator populations is a noteworthy and crucial subject to examine, Babu et al. (2022) [2], Kadhim et al. (2020) [3], Misra et al. (2017) [7], Misra et al. (2016) [5], Misra et al. (2016) [6], and Turner et al. (1997) [11].

Patchy distributions, in which certain populations, referred to as patches, have a higher population density than others, are synonymous with crowding. Ecological differential equation models have employed Lloyd's mean crowding index as a crowding indicator, Vallejos et al. (2020) [12], Subbey et al. (2020) [10], Waters (2014) [13]. The impact of crowding-specific viruses has been studied, and a model has been defined by the authors Pandey et al. (2022) [9]. They have provided a significant explanation and model interpretation. The main cause of the increasing COVID-19 case trend is crowding and many more. The impacts of overpopulation on the population as a whole can be predicted with the use of mathematical modeling. The prospective negative effects of crowding on the heart and lungs of adult albino rats, as well as the potential benefit of a combination sulpiride treatment, were examined by the authors of the research experiment by Amara et al. (2014) [1]. Only the rats that were subjected to overcrowding in the study displayed extremely detrimental alterations to their lungs, including thickening of the interalveolar septa and obliteration of the alveoli, infiltration of inflammatory cells within the pulmonary interstitium, fibrosis, and infiltration of the peribronchial, thickening of the walls of the pulmonary blood vessels, deposition of interstitial collagen fibers, and apoptotic cellular changes. At the heart level, the sulpiride category showed a substantial decline in the diameters of the cardiac strength fibers with focal areas of necrosis, apoptotic alterations, and increased deposition of collagen fibers.

In this paper, a food chain prey-predator model of biological example is considered and studied. This model is inspired by Manaqib et al. (2022) [4] and discusses the impact of toxic substances and the crowding effect on prey-predator populations. A qualitative analysis of the model is conducted, and numerical simulations are used to corroborate the findings.

2. The Mathematical Model

2.1. Assumptions

The assumptions that are considered to construct a food chain mathematical system with the effect of crowding and toxicants are given below:

The prey population has an intrinsic growth rate. The food chain is having the interactions between prey, intermediate predators, and top predators. To ensure its survival, the prey has the ability to create harmful substances. The population of top predators does not feed on the population of prey. Predation of prey determines the expansion of the predator population, while predation of intermediate predators determines the development of the top-last predator population. In populations of prey, intermediate predators, and top predators, intraspecific rivalry is taken into account. Migration does not exist as a consequence of population density, sometimes known as crowding effects.

Table 1 lists the variables and parameters together with the appropriate presumptions and definitions for the mathematical framework of the food chain.

Table 1. List of variables and parameters, and meanings.

Symbol	**Meaning**	**Type**	**Condition**
$X(t)$	The density of the prey population	Variable	$X(t) \geq 0$
$Y(t)$	The density of the intermediate predator population	Variable	$Y(t) \geq 0$
$Z(t)$	The density of the top predator population	Variable	$Z(t) \geq 0$
C_0	The intrinsic growth rate of prey	Parameter	$C_0 > 0$
C_1	The strength of competition among individuals of species prey	Parameter	$C_1 \geq 0$
C_2	The predation rate of prey population	Parameter	$C_2 \geq 0$
C_3	The rate which prey is capable to produce toxic substance for its survival	Parameter	$C_3 \geq 0$
C_4	The conversion rate	Parameter	$C_4 \geq 0$
C_5	The predation rate of intermediate predator population	Parameter	$C_5 \geq 0$
C_6	The natural death rate of intermediate predator	Parameter	$C_6 \geq 0$
C_7	The strength of competition among individuals of intermediate predator population	Parameter	$C_7 \geq 0$
C_8	The conversion rate	Parameter	$C_8 \geq 0$
C_9	The natural death rate of top predator	Parameter	$C_9 \geq 0$
C_{10}	The strength of competition among individuals of top predator population	Parameter	$C_{10} \geq 0$

2.2. Formulation of Model

A mathematical model of a food chain that takes the form of a system of nonlinear differential equations can be created based on certain assumptions:

$$\begin{aligned} \frac{dX}{dT} &= C_0X - C_1X^2 - C_2XY - C_3X^2Y \\ \frac{dY}{dT} &= C_4XY - C_5YZ - C_6Y - C_7Y^2 \\ \frac{dZ}{dT} &= C_8YZ - C_9Z - C_{10}Z^2 \end{aligned} \tag{2.1}$$

Using the following scaling transformations, we can decrease the number of parameters in the system (2.1) mentioned above:

$$X = \frac{C_0x}{C_1}, \quad Y = \frac{C_0^2y}{C_1C_2C_3}, \quad Z = \frac{C_0^3z}{C_1C_2C_3C_{11}}, \quad T = \frac{t}{C_0},$$
$$dX = \frac{C_0dx}{C_1}, \quad dY = \frac{C_0^2dy}{C_1C_2C_3}, \quad dZ = \frac{C_0^3dz}{C_1C_2C_3C_{11}}, \quad dT = \frac{dt}{C_0}.$$

Then system (2.1) becomes:

$$\begin{aligned}\frac{dx}{dt} &= x(1-x) - axy - bx^2y \\ \frac{dy}{dt} &= dxy - eyz - fy - gy^2 \\ \frac{dz}{dt} &= hyz - kz - jz^2\end{aligned} \tag{2.2}$$

$$a = \frac{C_0}{C_1C_3}, \qquad b = \frac{C_0^2}{C_1^2C_2}, \qquad d = \frac{C_4}{C_1}, \quad e = \frac{C_0^2}{C_1C_2C_{10}}, \quad f = \frac{C_6}{C_0},$$
$$g = \frac{C_7C_0}{C_1C_2C_3}, \quad h = \frac{C_0C_8}{C_1C_2C_3}, \quad k = \frac{C_9}{C_0} \quad j = \frac{C_0^2}{C_0C_2C_5}.$$

It is evident that the system (2.2) has continuous interaction functions and continuous partial derivatives on the subsequent positive three-dimensional spaces:

$$R_+^3 = \left\{(x,y,z) \in R_+^3 : x(0) \geq 0, y(0) \geq 0, z(0) \geq 0\right\}.$$

Therefore, the solution of the system (2.2) is unique and exists.

3. Model Analysis

We shall now locate the system's equilibrium points (2.2). The food chain model's equilibrium points can be found if $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$. The system (2.2) has following four non-negative equilibrium points in x, y, z space namely, $\hat{E}_0(0,0,0)$, $\ddot{E}_1(\ddot{x},0,0)$, $\tilde{E}_2(\tilde{x},\tilde{y},0)$ and $\bar{E}_3(\bar{x},\bar{y},\bar{z})$. We prove the existence of $\hat{E}_0$, $\ddot{E}_1$, $\tilde{E}_2$ and $\bar{E}_3$ as follows: The existence of $\hat{E}_0$ is obvious.

3.1. $\ddot{E}_1(\ddot{x},0,0)$ equilibrium point:

From the first equation of (2.2), we get $1-\ddot{x}=0$, that is

$$\ddot{x} = 1. \tag{3.1}$$

3.2. $\tilde{E}_2(\tilde{x},\tilde{y},0)$ equilibrium point:

From the first equation of (2.2), we get

$$\tilde{x} = 1 - a\tilde{y} - b\tilde{y}^2, \tag{3.2}$$

$\tilde{x} > 0$ if $1 > a\tilde{y} + b\tilde{y}^2$. From the second equation of (2.2) and (3.2), we get

$$A_1\tilde{y}^2 + A_2\tilde{y} - A_3 = 0 \tag{3.3}$$

where, $A_1 = db$, $A_2 = (ad+g)$ and $A_3 = d - f > 0$. The Equation (3.3) will always have a positive root $\tilde{y}$.

3.3. $\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$ equilibrium point:

From the first equation of (2.2), we get

$$\bar{x} = 1 - a\bar{y} - b\bar{y}^2, \tag{3.4}$$

$\bar{x} > 0$ if $1 > a\bar{y} + b\bar{y}^2$. From the third equation of (2.2), we get

$$\tilde{z} = \frac{h\bar{y} - k}{j}, \tag{3.5}$$

$\bar{z} > 0$ if $h\bar{y} > k$. Using (3.4) and (3.5), in second equation of (2.2), we get

$$B_1\bar{y}^2 + B_2\bar{y} - B_3 = 0 \tag{3.6}$$

where, $B_1 = dbj$, $B_2 = (ajd + eh + jg)$ and $B_3 = (dj + ek - jf) > 0$. The Equation (3.6) will always have a positive root $\bar{y}$.

3.4. Stability of Model

The system (2.2) aims to determine the stability of equilibrium by calculating the eigenvalues of the variational matrix concerning each equilibrium.

$$J(f(x,y,z)) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix}$$

where, $\frac{dx}{dt} = f_1(x,y,z)$, $\frac{dy}{dt} = f_2(x,y,z)$, and $\frac{dz}{dt} = f_3(x,y,z)$. The $J(f(x,y,z))$ elements are:

$$\frac{\partial f_1}{\partial x} = 1 - 2x - ay - by^2, \quad \frac{\partial f_1}{\partial y} = -ax - bxy, \quad \frac{\partial f_1}{\partial z} = 0,$$
$$\frac{\partial f_2}{\partial x} = dy, \quad \frac{\partial f_2}{\partial y} = dx - ez - f - 2gy, \quad \frac{\partial f_2}{\partial z} = -ey,$$
$$\frac{\partial f_3}{\partial x} = 0, \quad \frac{\partial f_3}{\partial y} = hz, \quad \frac{\partial f_3}{\partial z} = hy - k - 2jz.$$

Theorem 3.1. *The equilibrium point $\hat{E}_0$ is unstable.*

Proof. When we input the point of equilibrium $\hat{E}_0$ in $J(f(x,y,z))$, then the matrix $J(f(\hat{E}_0))$ is given by

$$J(f(\hat{E}_0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & -k \end{bmatrix}$$

and the characteristic equation is

$$(\lambda - 1)(f + \lambda)(\lambda + k) = 0.$$

Thus, the eigenvalues are $\lambda_1 = 1, \lambda_2 = -f$ and $\lambda_3 = -k$. Here, $\lambda_1 > 0$, therefore $\hat{E}_0 = (0,0,0)$ is unstable. ∎

Theorem 3.2. *If $\frac{1}{2} < \ddot{x}$ and $\ddot{x} < \frac{f}{d}$, then the point of equilibrium $\ddot{E}_1(1,0,0)$ is locally stable (asymptotically).*

Proof. When we input the point of equilibrium $\ddot{E}_1$ in $J(f(x,y,z))$, we obtain matrix $J(f(\ddot{E}_1)$ as

$$J(f(\ddot{E}_1)) = \begin{bmatrix} 1-2\ddot{x} & -a\ddot{x} & 0 \\ 0 & d\ddot{x}-f & 0 \\ 0 & 0 & -k \end{bmatrix}$$

and the characteristic equation is given by

$$(\lambda + 2\ddot{x} - 1)(\lambda + f - d\ddot{x})(\lambda + k) = 0.$$

Thus, the eigenvalues are $\lambda_1 = 1 - 2\ddot{x}$, $\lambda_2 = d\ddot{x} - f$ and $\lambda_3 = -k$. The point $\ddot{E}_1$ is locally stable (asymptotically) if $\lambda_1 < 0$ and $\lambda_2 < 0$ hold, *i.e.*,

$$\frac{1}{2} < \ddot{x}$$

and

$$\ddot{x} < \frac{f}{d}.$$

Therefore, $\ddot{E}_1$ is locally stable (asymptotically) if $\frac{1}{2} < \ddot{x}$ and $\ddot{x} < \frac{f}{d}$. ■

Theorem 3.3. *If $h\tilde{y} < k$, $p_1 > 0$, and $p_2 > 0$, then the equilibrium point $\tilde{E}_2(\tilde{x},\tilde{y},0)$ is locally stable (asymptotically), where $p_1 = a_1 + a_3 - d\tilde{x} - 1$, $p_2 = a_1a_3 + d\tilde{y}a_2 + d\tilde{x} - a_3 - a_1d\tilde{x}$, $a_1 = 2\tilde{x} + a\tilde{y} + b\tilde{y}^2$, $a_2 = \tilde{x}(a + b\tilde{y})$ and $a_3 = f + 2g\tilde{y}$.*

Proof. When the equilibrium point $\tilde{E}_2$ is substituted in $J(f(x,y,z))$, we get

$$J(f(\tilde{E}_2)) = \begin{bmatrix} 1-a_1 & -a_2 & 0 \\ d\tilde{y} & dx - a_3 & -e\tilde{y} \\ 0 & 0 & h\tilde{y}-k \end{bmatrix}$$

where, $a_1 = 2\tilde{x} + a\tilde{y} + b\tilde{y}^2$, $a_2 = \tilde{x}(a + b\tilde{y})$, $a_3 = f + 2g\tilde{y}$. Then the characteristic equation for $J(f(\tilde{E}_2))$ is given by

$$(h\tilde{y} - k - \lambda)(\lambda^2 + p_1\lambda + p_2) = 0, \tag{3.7}$$

where $p_1 = a_1 + a_3 - d\tilde{x} - 1$, $p_2 = a_1a_3 + d\tilde{y}a_2 + d\tilde{x} - a_3 - a_1d\tilde{x}$. Thus, the one eigenvalue is $\lambda_1 = h\tilde{y} - k < 0$, and other two eigenvalues will be considered by Routh-Hurwitz's criteria that $p_1 > 0$ and $p_2 > 0$ must be satisfied. Then by our assumption, $\tilde{E}_2$ is locally stable (asymptotically) *i.e.*,

$$h\tilde{y} < k,$$

$p_1 > 0$, and $p_2 > 0$. ■

Theorem 3.4. *The equilibrium point $\bar{E}_3(\bar{x},\bar{y},\bar{z})$ is locally asymptotically stable for the system (2.2).*

Proof. Let $\bar{E_3}$ be substituted in $J(f(x,y,z))$. We get

$$J(f(\bar{E_3})) = \begin{bmatrix} 1-m_1 & -m_2 & 0 \\ dy & dx-m_3 & -ey \\ 0 & hz & hy-m_4 \end{bmatrix}$$

where $m_1 = 2x + ay + by^2$, $m_2 = ax + bxy$, $m_3 = ez + f + 2gy, k + 2jz$. Then, the characteristic equation for $J(f(\tilde{E_3})$ is given by

$$\lambda^3 + M_1\lambda^2 + M_2\lambda + M_3 = 0 \tag{3.8}$$

where, $M_1 = m_1 + m_3 + m_4 - 1 - dx - hy$, $M_2 = m_1m_3 + m_1m_4 + m_3m_4 + m_2dy + heyz + dx + hy + dhxy - m_3 - m_4 - m_1dx - m_1hy - dxm_4 - m_3hy$, $M_3 = m_1m_3m_4 + m_1dhxy + m_1heyz + m_2m_4dy + m_4dx + m_3hy - dhxy - m_3m_4 - heyz - m_1m_4dx - m_1m_3hy - m_2dhy^2$. By Routh-Hurwitz's criteria, E_3 is locally stable (asymptotically) since the given conditions is satisfied

$$(m_1 + m_3 + m_4) > (1 + dx + hy),$$

$M_2 > 0$ *i.e.*, $m_1(m_3+m_4)+m_3m_4+dy(m_2+hx)+hy(1+ez)+dx > m_3(1+hy)+m_1(dx+hy)+m_4(1+dx)$, $M_3 > 0$ *i.e.*, $(m_1m_3m_4+m_1dhxy+m_1heyz+m_2m_4dy+m_4dx+m_3hy) > (dhxy + m_3m_4 + heyz + m_1m_4dx + m_1m_3hy + m_2dhy^2)$ and $M_1M_2 > M_3 > 0$.
Although it is challenging to understand the conclusions in ecological terms from the above complex expressions, graphs and numerical examples are used to show the system's dynamic behavior. ∎

4. Model Simulation

Matlab software is used to do numerical simulations. Each equilibrium point in the system (2.1) is simulated numerically, as well as simulations with various parameter modifications. The main purpose of the simulation is to check the dynamical behavior of prey-predator populations in the presence of toxicants and crowding, and also to verify the analytical results with numerical simulations. All the figures represent stability in the simulation.

Several authors have studied prey-predator mathematical numerical simulations by considering different parameter values for each equilibrium point [2, 5–7]. These parameter values are used for numerical simulations: $C_0 = 1.8; C_1 = 0.09; C_2 = 0.8; C_3 = 0.3; C_4 = 0.44; C_5 = 1; C_6 = 0.1; C_7 = 0.09; C_8 = 0.2; C_9 = 0.1; C_{10} = 0.21$; and the initial values are taken as $X(0) = 1; Y(0) = 1; Z(0) = 1$; and also the time range is taken from 0 to 200.

For $\hat{E}_0$, by increasing the rate of crowding of prey population ($C_1 = 0.09$ to 3894), then in $\hat{E}_0$, all populations will die out; the results are shown in Figure 1. The duration varied from 0 to 50. Figure 1 tells the instability of $\hat{E}_0$. That is, the ecosystem does not occur for this population.

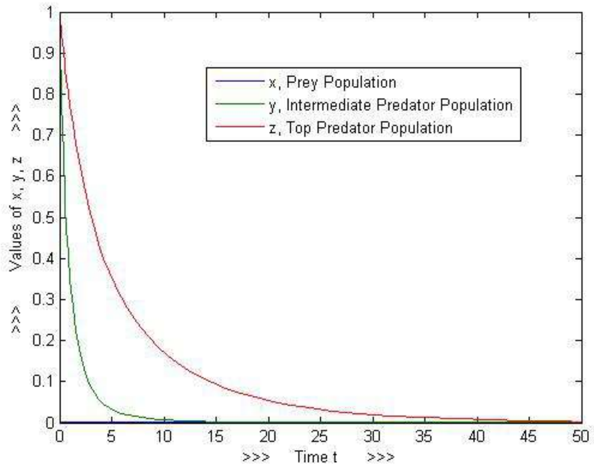

FIGURE 1. Instability behavior for $\hat{E}_0$.

For the equilibrium point $\ddot{E}_1$, only the prey population survives, whereas the predator population tends to become extinct with the given parameter values: $C_0 = 2; C_1 = 0.05; C_2 = 0.5; C_3 = 0.3; C_4 = 0.08; C_5 = 0.03; C_6 = 0.08; C_7 = 0.07; C_8 = 0.22; C_9 = 0.551; C_{10} = 0.981$, so the prey predator population is shown in Figure 2. The prey population increases *i.e.*, $\ddot{X} = 1$ in the absence of predator populations (see Table 2). All the conditions of Theorem 2 are verified by given parameter values.

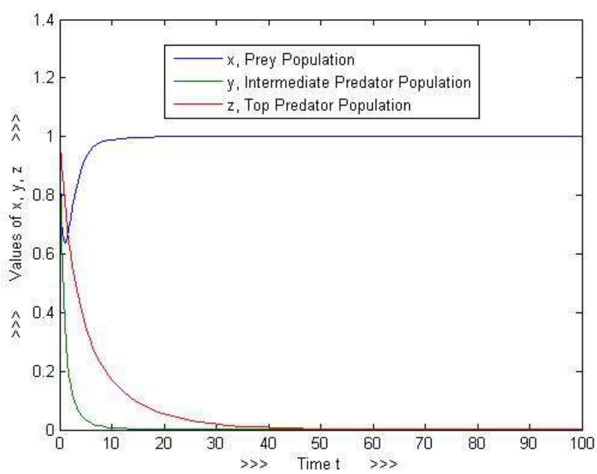

FIGURE 2. Stability behavior for $\ddot{E}_1$.

For the equilibrium point $\tilde{E}_2$, only prey and intermediate predator populations survive, whereas the top predator population tends to become extinct with the given parameter values: $C_0 = 2; C_1 = 0.05; C_2 = 0.5; C_3 = 0.3; C_4 = 0.08; C_5 = 0.03; C_6 = 0.08; C_7 = 0.07; C_8 = 0.22; C_9 = 0.551; C_{10} = 0.981$, so the prey predator population is shown in Figure 3. The populations *i.e.*, $\tilde{E}_2$ (2.3638, 1.5567, 0) in the absence of top predator populations (see table 2). All the conditions of Theorem 3 are verified by given parameter values.

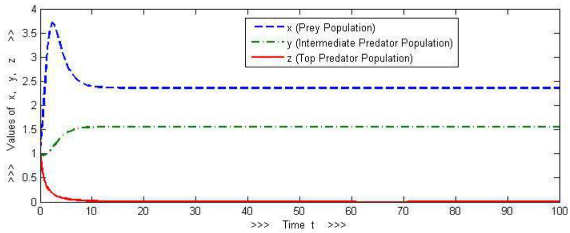

FIGURE 3. Stability behavior for $\tilde{E}_2$.

For the equilibrium point $\bar{E}_3$, all the populations would survive with the given parameter values: $C_0 = 1.8; C_1 = 0.09; C_2 = 0.8; C_3 = 0.3; C_4 = 0.44; C_5 = 1; C_6 = 0.1; C_7 = 0.09; C_8 = 0.2; C_9 = 0.1; C_{10} = 0.21$, so the prey-predator population is shown in Figure 4. The populations' values *i.e.*, $\bar{E}_3$(1.9337, 1.1770, 0.6454) (see table 2). All the conditions of Theorem 4 are verified by given parameter values.

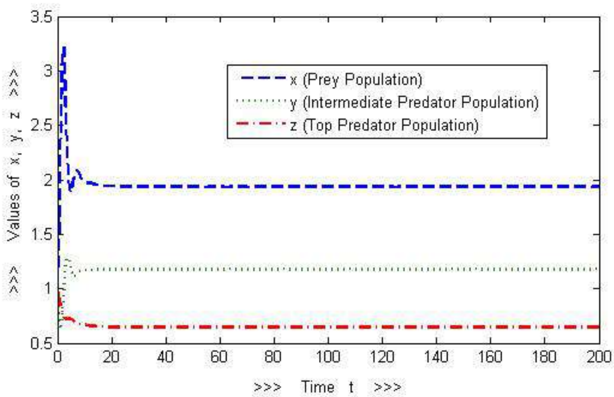

FIGURE 4. Stability behavior for $\bar{E}_3$.

For $\bar{E}_3$, the system (2.1) is verified by not considering toxicant. That means the system only studied crowding. It has been observed that all the prey and predator populations increased in values when we compared them with $\bar{E}_3$ values (see Table 2) and are stable. So the prey predators are shown in Figure 5, and the populations' values *i.e.*, $\bar{E}_3$(3.7115, 1.8329, 1.2690) (see table 1)(without toxicant case).

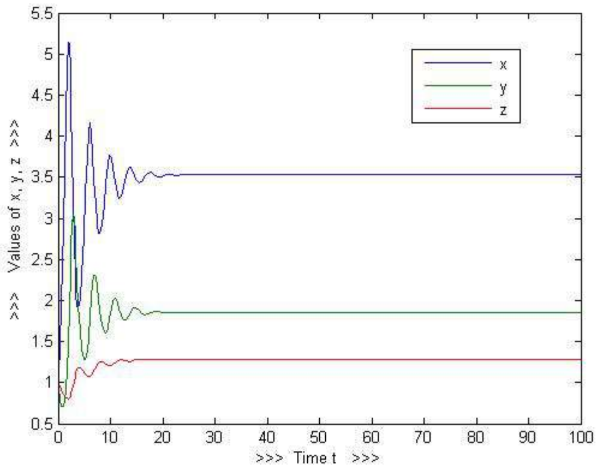

FIGURE 5. Stability behavior for $\bar{E}_3$ (without toxicant effect case).

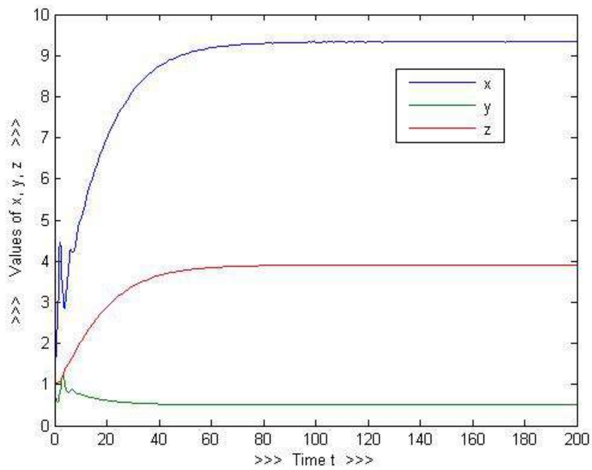

FIGURE 6. Stability behavior for $\bar{E}_3$ (without crowding effect case)

For $\bar{E}_3$, the system (2.1) is verified by not considering crowding. That means the system was only studied with the toxicant. It has been observed that all the prey and predator populations increased in values when we compared them with $\bar{E}_3$ values (see Table 2), and $\bar{E}_3$ is stable. So the prey predators are shown in Figure 6, and the populations' values *i.e.*, $\bar{E}_3(9.3347, 0.4998, 3.9067)$ (see table 1)(without crowding case).

For $\bar{E}_3$, the system (2.1) is verified by not considering crowding and toxicant. That means, the system (2.1) with $C_1 = C_3 = C_7 = C_{10} = 0$. It has been observed that the system (2.1) would be unstable and not exist. The system survives when either the toxicant effect is present, or the crowding effect is present in the system.

Table 2. x, y, z values at different equilibrium points

Equilibrium Points	**Values of** x	**Values of** y	**Values of** z
$\ddot{E}_1(\ddot{x}, 0, 0)$	1.0000	0.0000	0.0000
$\tilde{E}_2(\tilde{x}, \tilde{y}, 0)$	2.3638	1.5567	0.0000
$\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$	1.9337	1.1770	0.6454
$\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$ (without toxicant)	3.7115	1.8329	1.2690
$\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$ (without crowding)	9.3347	0.4998	3.9067
$\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$ (without toxicant and crowding)	unstable		

5. Conclusion

Considered a food chain mathematical which is based on the assumptions in the model, formulated with the effect of crowding and toxicant (see the system (2.1)). The considered prey-predator mathematical model has four equilibriums, they are $\hat{E}_0(0, 0, 0)$, $\ddot{E}_1(\ddot{x}, 0, 0)$, $\tilde{E}_2(\tilde{x}, \tilde{y}, 0)$ and $\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$. The survival of all the points was found, and the stability analysis was performed. The $\hat{E}_0(0, 0, 0)$ equilibrium was unstable, other points $\ddot{E}_1(\ddot{x}, 0, 0)$, $\tilde{E}_2(\tilde{x}, \tilde{y}, 0)$ and $\bar{E}_3(\bar{x}, \bar{y}, \bar{z})$ were locally asymptotically stable under some analytical conditions. We have used Matlab software for numerical simulations. All the figures show the stability in nature. With the help of parameter values, all the analytical results have been verified. The system shows stable cases in the absence of toxicant and crowding separately, but in the combined absence of toxicant and crowding, the system does not exist (unstable) (see Table 2). It has been observed that the system would survive when either the toxicant effect or the crowding effect is present in the system (see Figures 4 and 5).

References

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